A Reexamination of the Associations between Earnings Innovations, Persistence of Expected Earnings, Price-to-Earnings Ratios, and Earnings Response Coefficients

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Abstract
We (re)investigate the associations between earnings innovations, persistence of expected earnings, and the magnitudes of price-to-earnings ratios (P/E ratios) and earnings response coefficients (ERCs), respectively. In doing so, we extent the traditional price model by a simultaneous incorporation of three assumptions: (i) accounting earnings contain noisy components, (ii) earnings persistence is negatively correlated with the value-relevant magnitude of earnings innovations, and (iii) an ARIMA(1,1,0) process is appropriate in describing the time series properties of longitudinal earnings data. We first show on a theoretical level that under these assumptions: (i) P/E ratios and ERCs differ in their magnitudes and that it thus becomes inevitable to distinguish between these two constructs in designing price/return-earnings relations; (ii) ERCs are positively associated with the value-relevant magnitude of earnings innovations, extending prior findings in Freeman & Tse, 1992 and negatively associated with the persistence of expected earnings, extending prior findings in Kormendi & Lipe, 1987; (iii) built on our generalized price model and assuming a discount rate of 15%, we theoretically predict P/E ratios to range between 10-60 and ERCs to range between 0-6 within the earnings persistence interval of 0.6-1.0. We second test our model empirically. The results are consistent with our predictions, closing the gap between theoretically expected and empirically estimated P/E ratio and ERC magnitudes, respectively.

Keywords: Capital markets, earnings innovations, persistence of expected earnings, price-to-earnings ratio, earnings response coefficient.

JEL classification: M14, C20

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1. Introduction

Accounting theory uses standard price/return models to describe the associations between earnings innovations, earnings persistence, investors’ expectations, and the magnitudes of earnings response coefficients (ERCs) as well as current and expected price-to-earnings ratios (P/E ratios). The current stock price in the price model reflects the cumulative effect of earnings information and thus varies due to both expected and unexpected earnings (Kothari & Zimmerman, 1995, p. 156). The association between the current stock price and current expected earnings is measured by the expected P/E ratio and the association between the current stock price and unexpected earnings by the ERC.¹ Most often a ‘random walk’ is assumed to be appropriate in describing the time series properties of annual earnings (Ball & Brown, 1968; Ball & Watts, 1972; Kothari & Sloan, 1992). This assumption has at least two important implications. If annual earnings follow a random walk, i.e. an ARIMA(0,1,0)² process, then (i) earnings innovations are expected to be permanently persistent and (ii) to lead one-to-one to earnings revisions.³ As a consequence of the ‘random walk’ assumption, accounting theory predicts (i) current P/E ratios⁴ to equal expected P/E ratios; (ii) current expected P/E ratios to equal ERCs; and assuming a discount rate of 5-10% (iii) both P/E ratio and ERC magnitudes to range between 11 and 21.

Empirical evidence, however, only partially confirms these predictions. For instance, empirical estimates of ERCs lie significantly below the expected magnitudes of 11 to 21 (Kothari, 2001, p. 123-143). Kormendi & Lipe, 1987 show that earnings innovations are not totally persistent across firms

¹ Technically spoken, in a linear two-variable regression model where the dependent variable is the stock price and the independent variables are expected earnings and unexpected earnings, the slope coefficient capturing the association between the stock price and expected earnings is referred to as the (marginal) expected price-to-earnings ratio (P/E ratio) and the slope coefficient capturing the association between the stock price and unexpected earnings is referred to as the earnings response coefficient (ERC).
² ARIMA(p, d, q) denotes an autoregressive of the order p, integrated of the order d, moving average of the order q process.
³ That is, a one dollar earnings innovation triggers a one dollar revision in expected earnings.
⁴ Current P/E ratios reflect the association among the current stock price and current reported earnings.
and that ERC magnitudes are positively associated with the level of earnings persistence. Collins & Kothari, 1989 investigate, among other ERC determinants, the impact of earnings’ time series properties on ERC magnitudes. They find a random walk to be limiting in describing the time series properties of earnings and use instead an ARIMA(0,1,1) process to determine earnings persistence (p. 155). Ramakrishnan & Thomas, 1992 show that annual earnings are well described by a first-order autoregressive process. Beaver, Lambert, & Morse, 1980, Collins & Salatka, 1993, and Ramakrishnan & Thomas, 1998 show that reported earnings contain noisy and price-irrelevant components, decreasing ERC magnitudes. Based on the premise that the magnitude of earnings innovations is negatively associated with earnings persistence, Freeman & Tse, 1992 find ERCs to be non-linear functions. The mentioned studies provide important determinants of ERCs. However, they are limiting in terms of that they constitute isolated and thus partial explanatory approaches, constituting only parts of a wider complex price discovery process.

By bringing things together, we simultaneously incorporate the following three assumptions into the traditional price model. First, we introduce the ‘noise in earnings’ argument into the price model, i.e. earnings innovations are only partially value-relevant and investors are sophisticated enough to extract the value-relevant portion of them in equity valuation (Beaver et al., 1980). Second, we incorporate Freeman’s and Tse’s, 1992 premise of a negative association between the magnitude of earnings innovations and earnings persistence into the price model. Third, we assume an ARIMA(1,1,0) process to better explain the observed properties of longitudinal times series of annual earnings than a random walk. A simultaneous incorporation of these three hypotheses into the price model has significant economic implications for the associations between earnings innovations, earnings persistence, investors’ expectations, and the magnitudes of ERCs as well as current and expected P/E ratios. Further, it has also significant methodological implications for the design of price/return-earnings relations, leading to an innovative methodological refinement of the price model.
In contrary to the traditional price model which assumes that the strength of the stock price reaction due to expected and unexpected earnings, respectively, is of the same amount, i.e. expected P/E ratio equals the ERC, we first show on a theoretical level that if earnings contain noisy elements, it becomes inevitable to distinguish between P/E ratios and ERCs within the price model because the two constructs then differ in their magnitudes. Ignoring this fact will lead to biased estimates of P/E ratios and ERCs, respectively. In particular, in the context of the traditional price model it applies that the higher earnings persistence the higher the P/E ratio.\textsuperscript{5} This can easily be seen from an ARIMA(1,0,0) process.

The P/E ratio \((b)\) then amounts to: \(b = \left[1 + \frac{\phi}{1 + r - \phi}\right]\) where \(\phi\) denotes the level of earnings persistence and \(r\) the discount rate. Given a constant discount rate, \(b\) reaches its highest value if earnings are totally persistence, i.e. \(\phi = 1\). In this case the P/E ratio reduces to: \(b = \left[1 + \frac{1}{r}\right]\), which equals the P/E ratio under the random walk assumption.\textsuperscript{6} Further, since in the traditional price model the P/E ratio equals the ERC (i.e., both equal \(b\)), it also applies that the higher earnings persistence the higher the ERC. That is why prior literature refers to earnings persistence to be positively associated with ERC magnitudes. The results, however, can only be interpreted in this way under the restrictive assumptions of the traditional price model.

In our extended model results cannot be interpreted in this way anymore. First, the introduction of the ‘noise in earnings’ argument into the price model leads to the fact that P/E ratios (\(b\)) and ERCs differ in their magnitudes. In particular, the ERC then equals to the product of the P/E ratio and the value-relevant magnitude of an earnings innovation (\(\gamma\)): \(ERC = by\). Second, Freeman and Tse show that the value-relevant magnitude of an earnings innovation is negatively correlated with earnings persistence (\(\phi\)): \(\phi = f^-(\gamma)\). Third, if annual earnings follow an ARIMA(1,1,0) process, then the P/E ratio equals:

\textsuperscript{5} For the positive relation between ERCs and earnings persistence, see also the explanation in Kothari & Collins, 1989, p. 147.
\textsuperscript{6} The P/E ratio (\(b\)) under an ARIMA(1,0,0) process with \(\phi = 1\) equals the P/E ratio under an ARIMA(0,1,0) process, i.e. random walk.
\[ b = \left[ \frac{1}{\left( \frac{\gamma}{\gamma + 1} \right)} \right] \] and is thus a positive function of earnings persistence: \( b = f^+(\phi) \). Under these three assumptions, an increase in \( \gamma \) has two opposite effects on the ERC magnitude, namely the direct effect of an increase in \( \gamma \) itself and an indirect effect via the persistence of earnings \( (\phi = f^-(\gamma)) \) on \( b \) \( (b = f^+(\phi)) \) and in the end on the ERC. In summary, an increase in \( \gamma \) triggers two contrary effects on the ERC magnitude: \( \gamma \uparrow \Rightarrow \phi \downarrow \Rightarrow b \downarrow \Rightarrow ERC = \gamma(1)b(\uparrow) \). Thus it is an empirical question to test which effect dominates.

Our empirical findings show that the positive effect of an increase of \( \gamma \) outweighs its negative effect on the magnitude of the ERC. As a consequence, ERCs (P/E ratios) are negatively (positively) associated with earnings persistence and positively (negatively) with the value-relevant magnitude of earnings innovations, extending prior results. Our analysis shows that prior results, i.e. a positive association between earnings persistence and the ERC (Kormendi & Lipe, 1987; Collins & Kothari, 1989) as well as a negative association between the magnitude of the value-relevant fraction of earnings innovations and the ERC (Freeman & Tse, 1992) are only valid under the restrictive assumption of a random walk. Thus in cases where annual accounting earnings do not follow a random walk, results and economic conclusions inferred from this model may be biased. These implications have been overseen so far because (i) commonly used price/return models do not explicitly distinguish between expected P/E ratios and ERCs but rather assume both to be the same and (ii) mostly ignore the negative association between earnings persistence and the magnitude of the value-relevant fraction of earnings innovations in their research designs. We also show that it is reasonable to assume that longitudinal time series of annual earnings follow an ARIMA(1,1,0) process. Such time series are characterized by the fact that they have a trend and are in their original form not stationary. However, since the first differences of these times series are stationary, they are integrated of the order one (Gujarati & Porter, 2007, pp. 747, 776). If these properties apply to annual earnings, expected magnitudes of P/E ratios and ERCs change as compared to a random walk model. Expected magnitudes of P/E ratios then range approximately
between 10 and 60 and ERC magnitudes between 0 and 6, assuming a discount rate of 15% (Kothari & Sloan, 1992)\(^7\) and considering an earnings persistence interval of 0.6 to 1 (see figures 2 and 3). Our empirical findings are consistent with these magnitudes, closing the gap between theoretically expected and empirically estimated P/E ratio and ERC magnitudes, respectively.

Finally, we provide evidence on the determinants of nonlinearities of P/E ratios and ERCs. Freeman & Tse, 1992 assume ERCs to be non-linear functions due to a non-linear return-earnings relation, based on an arctangent function. In contrary, we do not a priori assume certain non-linear relations of ERCs and P/E ratios, respectively, but rather show that the P/E ratio is a non-linear positive (negative) function of the persistence parameter of expected earnings (magnitude of the value-relevant fraction of earnings innovations) and that the opposite applies to the ERC which is a non-linear negative (positive) function of the persistence parameter of expected earnings (magnitude of the value-relevant fraction of earnings innovations). Furthermore, our results show that disentangling the effects of expected and unexpected earnings on stock prices and including those separately in a two-variable regression model as well as allowing them to have different slope coefficients yields estimates on the two components that are closer to their predicted values and improves the explanatory power of the price model. Our results have also important implications for earnings quality and earnings informativeness research which uses ERCs as proxy variables for their respective construct. We show that earnings quality (informativeness) is high when P/E ratios are high and ERC are low.

Our paper proceeds as follows. Section 2 provides the extension of the price model and the derivation of the associations between earnings innovations, persistence of expected earnings, as well as the magnitudes of P/E ratios and ERCs, respectively, on a theoretical level. In doing so, we extend the traditional price model by (i) the ‘noise in earnings’ hypothesis, (ii) the premise that the magnitude of the value-relevant fraction of earnings innovations is negatively correlated with the persistence of expected earnings (‘negative \(\phi\cdot\gamma\)-relation’), and (iii) the assumption that an ARIMA(1,1,0) process is

\(^7\) Kothari & Sloan 1992, pp. 152-153 document an average realized annual rate of return of about 16-17%
appropriate in describing the time series properties of longitudinal earnings data. Section 2 also contains the development of hypotheses. In section 3, we describe the sample selection and operationalize the theoretical model for empirical estimation. Section 4 contains the results and section 5 additional robustness tests. Section 6 concludes.

2. Theoretical Background and Hypotheses Development

2.1 The Price Model

2.1.1 ERC Derivation under the Random Walk Assumption

Prior research often uses the price model to empirically estimate ERCs (Kothari & Zimmerman, 1995). In this context, the ERC is defined as the change in a firm’s stock price due to a one dollar unexpected earnings. In order to derive the magnitude of ERCs both price and return models start with a standard valuation model in which the stock price is the net present value of expected future cash flows per share. Assuming a one-to-one relation between net accounting earnings and net cash flows, as well as that earnings for a period contemporaneously reflect all the information that is incorporated in stock prices over the same period, i.e. prices do not lead earnings, yields the following valuation model:

\[ P_{i,t} = \sum_{s=1}^{T} \frac{X_{i,t+s}^e}{(1 + r)^s}, \quad s = 1, 2, \ldots, T \] (1)

where \( P_{i,t} \) is a firm’s stock price in time \( t \), \( X_{i,t+s}^e \) is expected net earnings per share, and \( r \) is the (constant) expected rate of return. Theory predicts that if unexpected earnings innovations are purely permanent, that is, if a one dollar earnings innovation induces a one dollar revision in expected earnings in all future periods \( (T \rightarrow \infty) \), the induced stock price change will equal:\[ P_{i,t} = X_{i,t}^e + X_{i,t+1}^e/(1 + r) + X_{i,t+2}^e/(1 + r)^2 + \cdots + X_{i,t+T}^e/(1 + r)^T. \]

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8 Kothari & Zimmerman, 1995 show that ERC estimates from price models are economically less biased, that is, are closer to their theoretically predicted values as compared to return models. We therefore build our analysis primary on the price model instead on any type of return models.

9 This is the case when time series of accounting earnings can be described by a random walk model.

10 Why the ERC corresponds to the expression in equation (2) can easily be seen when equation (1) is disassembled into its individual components: \( P_{i,t} = X_{i,t}^e + X_{i,t+1}^e/(1 + r)^1 + X_{i,t+2}^e/(1 + r)^2 + \cdots + X_{i,t+T}^e/(1 + r)^T \).
Thus, under the made assumptions the association between stock prices and expected future earnings is an inverse function of the expected rate of return \( r \), which is the sum of the risk-free rate of return and a risk premium. Model (1) then transforms to:

\[
P_{i,t} = a + b X_{i,t+1}
\]

where \( X_{i,t+1} \) denotes a firm’s \( i \) expected earnings in period \( t \) for the period \( t + 1 \) conditional on all information incorporated in current and past earnings at time \( t \), \( a \) is an intercept, and \( b \) is a slope coefficient, capturing the association between the stock price and expected future earnings (marginal expected P/E ratio)\(^{11} \) corresponding to: \( b = 1 + 1/r \). Since market’s expectations of future net earnings are unobservable, it is necessary to derive a reliable proxy for expected earnings in order to be able to estimate the ERC from equation (3). One common assumption in this context is to presume that a random walk, i.e. ARIMA(0,1,0), is a reasonable description of the time series properties of reported earnings \( (X) \).\(^{13} \)

\[
X_{i,t} = X_{i,t-1} + \epsilon_{it}
\]

A special feature of the random walk model is the persistence of random shocks (Maddala & Lahiri, 2009, p. 555). That is, occurred shocks have an infinite impact on \( X_{i,t} \) and never die away. Further, current earnings reflect both a ‘stale’ component of earnings which had been anticipated by the market \( (X^e) \) and an earnings surprise \( (X^u) \) which had not been anticipated and thus conveys new information to the market in period \( t \):

\[
X_{i,t} = X_{i,t}^e + X_{i,t}^u
\]

---

\(^{11}\) In particular, since \( b \) measures the association between the current stock price and one-year ahead expected earnings, it can be interpreted as the marginal expected one-year forward P/E ratio.

\(^{12}\) For the proof of this relation, see e.g. the appendix in Kothari & Zimmerman, 1995.

\(^{13}\) The random walk model states that the value of reported accounting earnings in one period equals its value in the previous time period plus a random unexpected shock, \( \epsilon_{it} \sim IN(0, \sigma^2) \).
Combination of the random walk assumption with the identity equation (5) provides an estimate for the unobservable variable of expected earnings in (3):

\[ X_{i,t+1}^e = X_{i,t} \]  

(6)

This type of expectation formation is called naïve expectations and was widely used in early economics literature in order to operationalize expectation formation. Because of the relation stated in equation (6), equation (7) represents the empirical equivalent to equation (3):

\[ P_{i,t} = a + bX_{i,t} \]  

(7)

where \( X_{i,t} \) is reported net earnings per share of firm \( i \) in time \( t \). Substitution of equation (5) into equation (6) yields:

\[ X_{i,t+1}^e = X_{i,t}^e + X_{i,t}^u \]  

(8)

Equation (8) shows that under the random walk assumption of reported earnings, expected earnings also follow a random walk. Future expectations therefore depend on what had been expected for the current period plus a current random shock. Differentiation of equation (8) with respect to \( X_{i,t}^u \) leads to:

\[ \partial X_{i,t+1}^e / \partial X_{i,t}^u = 1 \]  

(9)

14 Assuming that earnings innovations in equation (5) equal the random shocks in equation (4), that is \( X_{i,t}^u = \epsilon_{i,t} \), and subtracting equation (5) from (4) as well as rearranging terms, leads to expression (6): \( X_{i,t+1}^e = X_{i,t} \).

15 The major advantage of the random walk assumption is that current earnings can directly be used as a proxy for expected future earnings and thus for the estimation of ERCs without the need of modeling a specific investors’ expectation forming behavior. However, the major drawback of this type of expectations is that it is based on weak economic theory.


17 Within the traditional price model, the current P/E ratio in (7) and the expected one-year forward P/E ratio in (3) are of the same amount, namely \( b \).

18 It is important to emphasize that this assumption implies that the ‘real’ degree of persistence in earnings innovations, as measured by equation (4), exactly corresponds to the perceived degree of persistence in expected earnings, as measured by (8). In practice, however, the persistence of reported earnings may differ from investors believes about the degree of persistence as measured by the persistence of expected earnings. Since it is the latter that is relevant in equity valuation, and the persistence of reported earnings is only an approximation for this unobservable construct, a gap between the two concepts may lead to biased results in the assessment of stock price reactions.
An increase of unexpected earnings by 1 dollar induces future expectation revisions of the same amount. Substitution of (8) in (3) and multiplying out, leads to:

\[ P_{i,t} = a + bX_{i,t}^e + bX_{i,t}^u \]  

Equation (10) shows that the current stock price reflects the cumulative effect of earnings information, and thus varies due to both the surprise and the stale component (Kothari & Zimmerman, 1995). The impact of the respective variable on the stock price is measured by the coefficient \( b \). Under the random walk assumption of reported earnings both the marginal price-to-earnings ratio (P/E ratio), measuring the relation between the stock price and current expected earnings:

\[ \frac{\partial P_{i,t}}{\partial X_{i,t}^e} = b \]  

as well as the ERC, capturing the relation between the stock price and current unexpected earnings:

\[ \frac{\partial P_{i,t}}{\partial X_{i,t}^u} = b \]  

are of the same amount, namely \( b \), and equal to: \( 1 + 1/r \). In this case there is no reason to distinguish conceptually between the two coefficients. Assuming an expected rate of return in the range of 5-10% leads to a theoretical magnitude of the marginal P/E ratio and of the ERC of about 11-21.

**Proposition 1.** Assume that model (10) is the true model and all assumptions made in its derivation hold, then the marginal P/E ratio in (11) equals the ERC in (12) and estimates of the coefficient \( b \) derived from model (7), will range between 11-21, assuming a discount rate of 5-10%.

**Proof.** Follows directly from (4), (7), and (10).

### 2.1.2 Noisy Accounting Earnings

The random walk assumption faces the problem that it is restrictive and thus will usually not hold in practice. From a theoretical point of view it is unlikely that in any given point in time investors con-
sider the whole earnings surprise as permanently value-relevant. This assumption implies that investors are naïve, viewing every single earnings innovation entirely as value-relevant. Prior literature examines price-earnings relations under the assumption of noisy earnings and shows that ERCs positively vary with the amount of the value-relevant component of an earnings innovations (Landsman & Magliolo, 1988, model 3, pp. 598-599; Beaver et al., 1980; Ramakrishnan & Thomas, 1998). Thus, a more realistic assumption is to presume that only a certain portion of an earnings surprise will be viewed as permanently value-relevant and will lead to investors’ expectation revisions. This assumption implies that investors are sophisticated enough to carefully analyze earnings innovations and to identify their value-relevant portion. If \( \gamma \) denotes the fraction of unexpected earnings that investors believe to be value-relevant, the expectation formation in (8) can be transformed to:

\[
X_{t+1}^e = X_t^e + \gamma X_t^u
\]  

(13)

Equation (13) differs from equation (8) by the fact that only the fraction \( \gamma \) of unexpected earnings is viewed as permanently value-relevant, having an impact on stock prices. Equation (13) therefore constitutes a modified random walk model. Now, revisions in expectations due to a one dollar earnings innovation are given by the differentiation of equation (13) with respect to \( X_t^u \):

\[
\frac{\partial X_{t+1}^e}{\partial X_t^u} = \gamma
\]  

(14)

Compared to equation (9) not the entire innovation is incorporated in future earnings expectations but rather only the portion \( \gamma \). Substituting equation (13) into equation (3) and multiplying out, leads to:

\[
\Delta X_{t+1}^e = \gamma X_t^u
\]

---

20 Where \( \gamma \), such that \( 0 < \gamma \leq 1 \), see e.g. Gujarati & Porter, 2007, p. 630.

21 Since the gap between current and expected earnings in time \( t \) is nothing else but unexpected earnings \( (X_t^u = X_t - X_t^e) \), equation (13) can be transformed to: \( X_{t+1}^e = X_t^e + \gamma (X_t - X_t^e) \). If \( \gamma = 1 \), earnings innovations are entirely value-relevant and purely permanent, leading to expectation revisions by the same amount. A value of \( \gamma \) of 1 implies that \( X_{t+1}^e = X_t^e \) which is the same as the expression in equation (6). Thus the modified random walk hypothesis includes as a special case (\( \gamma = 1 \)) the same economic implications for investors’ expectation formation as the random walk assumption of reported earnings. Reported accounting earnings then can be used as a proxy for expected future earnings. If \( \gamma = 0 \), earnings innovations have zero value-relevance and do not lead to any changes in expectations. In this case investors do not change their expectations due to any randomly occurring shocks, \( X_{t+1}^e = X_t^e \), meaning that expectations are ‘static’ and conditions today about expected earnings will be maintained in subsequent periods, irrespective of any earnings innovations. Reported accounting earnings then cannot be used as a proxy for expected future earnings. In contrary to the two extreme cases, the probably most realistic one is the assumption of partially value-relevant earnings innovations, \( 0 < \gamma \leq 1 \).

22 In discrete notation revisions in expectations are given by transformation of equation (13) to: \( \Delta X_{t+1}^e = \gamma X_t^u \).
\[ P_{t,t} = a + bX_{t,t}^e + b\gamma X_{t,t}^u \]  

(15)

The differentiation of (15) with respect to expected earnings, leads to a marginal expected P/E ratio which is of the same amount as in (11):

\[ \frac{\partial P_{t,t}}{\partial X_{t,t}^e} = b \]  

(16)

Now, however, the differentiation of (15) with respect to unexpected earnings, gives an ERC of:

\[ \frac{\partial P_{t,t}}{\partial X_{t,t}^u} = b\gamma \]  

(17)

which differs from (12). In particular, the ERC is now the product of the marginal P/E ratio, capturing the relation between the stock price and current expected earnings \( b \) and the value-relevant portion of unexpected earnings \( \gamma \).

**Proposition 2.** Assume that model (15) is the true model, expectations are formed as stated in equation (13), and discount rates range between 5-10% then: (i) the theoretical magnitude of the marginal P/E ratio, \( b \), will range between 11-21; (ii) the theoretical magnitude of the ERC derived from model (15) will be to the amount of \( \gamma \) lower than the marginal P/E ratio \( b \) if \( \gamma < 1 \); (iii) the theoretical magnitude of the ERC derived from model (15) will be smaller than the ERC magnitude derived from model (10).

**Proof.** Follows directly from the assumptions made and equations (3), (4), (10), (13) and (15).

The introduction of the ‘noise in earnings’ argument into the price model has important implications for the estimation of ERCs. As shown in (15) it becomes necessary to distinguish between the marginal P/E ratio \( b \) on the one hand and the ERC \( b\gamma \) on the other hand. This fact has been ignored in prior literature, where the marginal P/E ratio and the ERC have been treated as the same.

2.1.3 Transitory Earnings and Nonlinearities

As can be seen from (8) and (13), in both the pure and the modified random walk model, future expected earnings are a function of current expected and unexpected earnings. Both models assume purely persistent earnings innovations.\(^{23}\) In order to obtain a more general model, we refine investors’ ex-

\(^{23}\) The autoregressive coefficient in (8) and (13) is assumed to equal one.
pection formation by taking into account that earnings innovations do not necessarily have to be purely permanent (Collins & Kothari, 1989). As a consequence, by now expected earnings follow an autoregressive process of the order one:

$$ X_{i,t+1}^e = \phi X_{i,t}^e + \gamma X_{i,t}^u $$

where $\phi$ denotes the degree of the persistence of earnings innovations in expected earnings and $\gamma$ the value-relevant portion of an earnings innovation. Further and similar to Freeman & Tse, 1992, we take into account that a negative relation exist between $\gamma$ and $\phi$. Freeman & Tse, 1992 find that the absolute magnitude of value-relevant unexpected earnings is negatively correlated with earnings persistence. In a similar vein, we argue that the lower the unexpected but in retrospect proven to be value-relevant component of an earnings surprise (as a percentage of total earnings surprises, i.e. $\gamma$) the higher the persistence of expected earnings ($\phi$). The theory behind this link is as follows: The lower the in retrospect proven to be value-relevant fraction of investors’ forecast error the lower their expectation revisions. A low level of the need for expectation revisions implies a high level of earnings informativeness prior to the earnings announcement. Investors who were well-informed about the value-relevant portion of reported earnings prior to the earnings announcement will in retrospect (i.e. after the earnings announcement) not have to revise their expectation at all, or only to a very low amount. Thus, earnings informativeness is negatively associated with $\gamma$, i.e. the ‘revision coefficient’. Accounting theory in turn assumes that high earnings informativeness is positively associated with high earnings persistence, in our context, with high persistence of expected earnings ($\phi$). If earnings informativeness is high, implying high earnings persistence ($\phi \rightarrow 1$) investors are well-informed about a firm’s future economic performance and the price-relevant component of reported earnings. In this case investors will not have to revise their expectation after an earnings announcement at all ($\gamma \rightarrow 0$). In contrary, if investors revise their expectations each period by the full amount of the forecast error ($\gamma \rightarrow 1$), earnings informativeness regarding the price-relevant component of reported earnings and a firm’s future performance is very low, implying low earnings persistence ($\phi \rightarrow 0$). Thus, a negative
correlation between \( \gamma \) and \( \phi \) is likely to exist. A sufficiently general function covering this negative relationship is (see figure 1):\(^{24}\)

\[
\phi = \alpha_0 - \gamma^\alpha_1
\]  
\[(19)\]

It seems reasonable to assume that \( \alpha_0 = 1 \) and \( \alpha_1 = 1 \). In this case \( \phi \) becomes a linear function of \( \gamma \).

\[
\phi = 1 - \gamma
\]  
\[(20)\]

If the negative relation (20) is existent, time series properties of expected earnings change as compared to (8) and (13). In particular, if \( \phi = 1 \) then \( \gamma = 0 \) and equation (18) transforms to: \( X_{i,t+1}^e = X_{i,t}^e \). That is, expectations are purely deterministic based on their own past values. Investors then believe that conditions today about expected earnings will be maintained in subsequent periods. For example, investors expect a firm’s earnings to be constant at a level of 100 $ or to grow at a constant rate of 2%. In this case future expected earnings are perfectly predictable. In contrary, if \( \phi = 0 \) then \( \gamma = 1 \) and equation (18) transforms to: \( X_{i,t+1}^e = X_{i,t}^u \). Under the assumption that unexpected earnings occur randomly \( X_{i,t}^u \sim \mathcal{IN}(0, \sigma_{X_u}^2) \), investors’ expectation formation is a purely stochastic process. In this case future expected earnings are unpredictable. For any other combination of \( 0 < \phi < 1 \) and \( 0 < \gamma < 1 \) investors’ expectations follow the time series process stated in (18) and future earnings expectations are a weighted average of current expected and unexpected earnings. Substitution of (18) in (3), leads to:

\[
P_{i,t} = a + b \phi X_{i,t}^e + b \gamma X_{i,t}^u
\]  
\[(21)\]

In contrary to (11) and (16), the marginal P/E ratio is now given by:

\[
\partial P_{i,t} / \partial X_{i,t}^e = b \phi
\]  
\[(22)\]

The ERC is as in (17) still given by:

\[
\partial P_{i,t} / \partial X_{i,t}^u = b \gamma
\]  
\[(23)\]

Considering equations (22) and (23) it becomes clear that under the discussed assumption it is inevitable to distinguish between P/E ratios and ERCs within the price model.

\(^{24}\) Figure 1 shows different pathways of equation (19) depending on the magnitude of \( \alpha_1 \).
2.1.4 Times Series Properties and Expected Magnitudes of P/E ratios and ERCs

Since most prior literature investigating the determinants of ERCs is focused on explanations of its cross-sectional variation, relatively short time series of earnings are used for the empirical analyses (3-5 consecutive years). In this context assuming a random walk to describe the time series properties may seem to be appropriate because often relatively short time series are not characterized by a long-term trend. In contrary, we focus on longitudinal earnings data (15 consecutive observations).\textsuperscript{25} Such time series are usually characterized by the fact that they are non-stationary, having a deterministic trend (Gujarati & Porter, 2007, p. 745-747). However, the first differences of most of such time series are stationary and they are said to be integrated of order one. As mentioned above, empirical evidence also shows that annual earnings are well described by a first-order autoregressive process (Ramakrishnan & Thomas, 1992). Taking things together, some evidence exist suggesting that longitudinal earnings data is likely to follow an ARIMA(1,1,0) process. If expected earnings follow an ARIMA(1,1,0) process, the P/E ratio, \( b \), in equation (3) equals:\textsuperscript{26}

\[
b = \left[ \frac{1}{\left( \frac{r}{1+r} \right) \left[ 1 - \frac{\phi}{1+r} \right]} \right]
\] (24)

and thus, is a positive function of the persistence parameter \( \phi \), (Collins & Kothari, 1989, p. 148). Substitution of \( b \) in equation (22) by expression (24) leads to:

\[
\frac{\partial P_{t,t}}{\partial X_{i,t}^e} = b \phi = \left[ \frac{\phi}{\left( \frac{r}{1+r} \right) \left[ 1 - \frac{\phi}{1+r} \right]} \right]
\] (25)

\textsuperscript{25} Kothari & Zimmermann, 1995 provide some evidence on estimated ERC magnitudes from the price model based on longitudinal time series date of 20 consecutive years.

\textsuperscript{26} For each ARIMA\((p,d,q)\) process the ERC is given by: \( ERC = \frac{1 - \sum_{k=1}^{d} \left( \frac{r}{1+r} \right)^k \theta^s}{\left( \frac{r}{1+r} \right)^d \left[ 1 - \sum_{j=1}^{p} \left( \frac{r}{1+r} \right)^j \phi_j \right]} \) where \( \phi_j \) is an autoregressive coefficient of order \( j \); \( \theta^s \) is a moving average coefficient of order \( s \); \( r \) is the discount rate (cost of capital).
When expectations are formed as stated in (18), the marginal P/E ratio is a positive non-linear function of the persistence of expected earnings (\(\phi\)) (see figure 2). The ERC in (23) can also be expressed as a function of \(\phi\):\(^{27}\)

\[
\frac{\partial P_{i,t}}{\partial x_{i,t}^u} = b\gamma = \left[ \frac{1 - \phi}{r + \frac{\phi}{1 + r}} \right] \frac{1 - \phi}{1 + r} \tag{26}
\]

Equation (26) shows the ERC as a negative non-linear function of the persistence parameter \(\phi\). Figure 2 also provides early indication of the expected magnitudes of P/E ratios and ERCs, based on the discussed assumptions. Assuming a discount rate of 15%\(^{28}\) and considering the persistence interval of 0.6 to 1, P/E ratios range between 10 and 60; ERCs range between 6 and 0. Figure 3 shows the P/E ratio and the ERC as functions of \(\gamma\).

**Proposition 3.** Assume that model (21) is the true model, expectations are formed as stated in equation (18), and a negative association between the persistence of expected earnings and the value-relevant fraction of unexpected earnings exist as stated in (20), then: (i) the marginal P/E ratio (ERC) is a positive (negative) non-linear function of the persistence coefficient \(\phi\); (ii) the marginal P/E ratio (ERC) is a negative (positive) non-linear function of the value-relevant fraction of unexpected earnings \(\gamma\); (iii) assuming a discount rate of 15%, marginal P/E ratios (ERCs) from (21) will range between 10-60 (6-0), within the interval \(0.6 \leq \phi \leq 1\).

**Proof.** Follows directly from (20), (21), (25) and (26).

Our analytical analysis provides evidence on the determinants of the nonlinearity of marginal P/E ratios and ERCs. In particular, we contribute to prior research by providing determinants for the ‘S-shaped’ relations between stock prices and earnings first reported by Freeman & Tse, 1992. We show that the non-linear relations arise from the way how earnings persistence and the value-relevant por-

\(^{27}\) Substitution of \(b\) in (23) by (24) and of \(\gamma\) in (23) by (20) leads to equation (26).

\(^{28}\) Since \(\phi\) is a linear negative function of \(\gamma\), namely \(\phi = 1 - \gamma\), both equations (25) and (26) can also be expressed as functions of \(\gamma\). Equation (25) then transforms to: \(\frac{\partial P_{i,t}}{\partial X_{i,t}^u} = b\phi = \left[ \frac{1 - \gamma}{r + \frac{\gamma}{1 + r}} \right] \frac{1 - \gamma}{1 + r}\) and equation (26)

\(\frac{\partial P_{i,t}}{\partial X_{i,t}^u} = b\gamma = \left[ \frac{\gamma}{r + \frac{\gamma}{1 + r}} \right].\)

\(^{29}\) Kothari & Sloan 1992, pp. 152-153 document an average realized annual rate of return of about 16-17 %.
tion of earnings innovations, respectively, determine the magnitudes of marginal P/E ratios and ERCs. Equation (27) shows the way how earnings persistence $\phi$ determines the marginal P/E ratio as well as the ERC:30

$$P_{lt} = a + \left[ \frac{\phi}{(1 + r)} \right] X_{lt}^e + \left[ \frac{1 - \phi}{(1 + r)} \right] X_{lt}^u$$

Equation (28) shows how $\gamma$ determines the two coefficients within the price model: 31

$$P_{lt} = a + \left[ \frac{1 - \gamma}{(1 + r)} \right] X_{lt}^e + \left[ \frac{\gamma}{(1 + r)} \right] X_{lt}^u$$

Figure 4 replicates the findings of Freeman & Tse, 1992. Comparing results presented in figure 3 with the results in figure 4 reveals that (i) the functional form of the nonlinearity in ERCs first find by Freeman & Tse, 1992 can be explained by equation (25) and that (ii) it rather applies to the marginal P/E ratio than to the ERC. This fact has been overseen because prior research designs do not explicitly distinguish between marginal P/E ratios and ERCs.

2.2 Hypotheses Development

Kormendi & Lipe, 1987 show that ERCs are positively correlated with earnings persistence. Collins & Kothari, 1989; Ramakrishnan & Thomas, 1992; discuss time series properties of reported earnings and provide evidence that reported earnings do not necessarily follow a random. Instead different types of ARIMA processes seem to be more appropriate in describing the time series properties of annual earnings (Brooks & Buckmaster, 1976; Brown, 1993). Freeman & Tse, 1992 show that the ERC is negatively associated with the absolute magnitude of unexpected earnings and that this result is based on the premise that the absolute value of unexpected earnings is negatively correlated with earnings persistence (‘negative $\phi$-$\gamma$-relation’). They further show that the ERC is a non-linear function of the absolute magnitude of unexpected earnings. All mentioned results, however, are obtained from research.

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30 In order to obtain equation (27), substitute equations (25) and (26) into equation (21).
31 In order to obtain equation (28), substitute the expressions for P/E ratio $= b\phi$ and ERC $= b\gamma$ denoted in footnote 23 into equation (21).
designs, either based on price or return models which do not explicitly distinguish between marginal
P/E ratios and marginal responses of stock prices to unexpected earnings (ERCs). As shown in our
analytical analysis, this distinction is necessary when earnings contain noisy components and investors
are sophisticated enough to identify the value-relevant component of an earnings innovation and to
price it. Further, if a negative relation exists between the magnitude of the value-relevant portion of
earnings innovations and the persistence of expected earnings, the association between earnings inno-
vations, persistence of expected earnings, and the magnitudes of the marginal P/E ratio and the ERC,
respectively, change as compared to prior findings.

In particular, under these conditions ERCs are negatively associated with the persistence of expected
earnings, extending prior findings in Kormendi & Lipe, 1987; Collins & Kothari, 1989 and positively
associated with the magnitude of the value-relevant portion of earnings innovations, extending find-
ings in Freeman & Tse, 1992. P/E ratios, however, remain positively associated with persistence of
expected earnings and negatively with magnitude of the value-relevant portion of earnings innova-
tions, being consistent with prior findings.

To see why results change, it is necessary to take a closer look at the mechanisms behind model (21) in
conjunction with equation (20). The higher the permanent component of earnings surprises as a per-
centage of total earnings surprises ($\gamma$), the lower is the persistence of expected earnings ($\phi$), this fol-

\begin{equation}
\phi = \frac{1}{(1+\gamma)} \left[ \frac{\phi}{1+\gamma} \right] \cdot \frac{1}{(1+\gamma)}
\end{equation}

Further, since $b$ is a positive function of the persistence of expected earnings as stated
in (24): $b = \frac{\gamma}{(1+\gamma)} \left[ \frac{\phi}{1+\gamma} \right]$, it decreases as $\gamma$ increases. Given that the ERC equals the product $by$, two
opposite effects simultaneously have an impact on its magnitude, namely the direct effect of an in-
crease in $\gamma$ itself and an indirect effect via the persistence of earnings ($\phi = f^{-}(\gamma)$) on $b$ ($b = f^{+}(\phi)$)
and in the end on the ERC. In summary, the following line of arguments exist in our model: $\gamma \uparrow \Rightarrow \phi \downarrow
\Rightarrow b \downarrow \Rightarrow ERC = by \downarrow \uparrow$. It can be shown that under assumption (20), i.e. a linear negative relation
among $\phi$ and $\gamma$ as well as the assumption of a constant rate of return the positive effect of an increase
in $\gamma$ outweighs the negative one on the ERC magnitude, leading to two at first appearance contradicting results compared with prior findings. First, the magnitude of ERCs is now negatively correlated with the persistence of expected earnings and second, the ERC is positively correlated with the value-relevant fraction of earnings innovations. These at first appearance ‘contradicting’ results arise in the end from the facts that (i) prior research does not distinguish between the marginal P/E ratio and the ERC in designing price/return-earnings relations and (ii) mostly ignores the negative relation among $\phi$ and $\gamma$ in the estimation of price/return-earnings relations.

Considering the relations between the marginal P/E ratio and the persistence of expected earnings on the one hand as well as the value-relevant portion of earnings innovations on the other hand, results remain consistent with prior findings. Since prior research does not distinguish among P/E ratios and ERCs, in summary, the following line of arguments has been assumed so far: $\gamma \uparrow \Rightarrow \phi \downarrow \Rightarrow b \downarrow = ERC \downarrow$. In particular, since the marginal P/E ratio, $b$, equals the ERC in the traditional price model, both are positively associated with earnings persistence and negatively with the value-relevant fraction of earnings innovations. However, if reported earnings contain noisy components and $\gamma$ and $\phi$ are negatively correlated, this argumentation rather applies to P/E ratios than to ERCs. We therefore distinguish between the two constructs and hypothesize:

**H1:** The ERC (P/E ratio) is negatively (positively) associated with the persistence of expected earnings ($\phi$).

**H2:** The ERC (P/E ratio) is positively (negatively) associated with the value-relevant fraction of an earnings innovations ($\gamma$).

Finally, the distinction between the marginal P/E ratio and the ERC is necessary in order to obtain unbiased estimates of the two coefficients. Since it is likely that the two coefficients will have different magnitudes, i.e. $\phi \neq 1$ and $\gamma \neq 1$ in equation (21), disentangling expected and unexpected earnings components from reported earnings and including them separately in the regression will yield esti-
mates on the two components that are closer to their predicted values. As a consequence, the estimated P/E ratio and the ERC will be unbiased and the model’s explanatory power will increase.

The question of main interest is: what are appropriate predictions for these coefficients? As discussed before, prior literature assumes magnitudes for P/E ratios and ERCs of about 11-21, based on discount rates of 5-10%. These magnitudes, however, are based on very restrictive assumptions. Taking into account that (i) expected earnings rather follow an ARIMA(1,1,0) process and that (ii) the value-relevant magnitude of earnings innovations and the persistence of expected earnings are negatively correlated, requires revisions about the magnitudes of P/E ratios and ERCs. Based on equation (25) and (26) as well as assuming (i) the persistence of expected earnings to range between 0.6 to 1 as well as (ii) a discount rate of about 15%, we hypothesize:

\[ H3: \text{Estimates of P/E ratios (ERCs) derived from model (21) will range within the interval } 0.6 \leq \phi \leq 1 \text{ between 10-60 (0-6).} \]

3. Empirical Analysis

3.1 Data and Sample Selection

The initial sample consists of all unique firm-year observations available in year 2013 on the Compustat North America Annual Industrial database file covering the period from 1962–2013 for the stock price (334,190 firm-year obs.) and earnings per share (342,134 firm-year obs.). In particular, the two variables are the annual close stock price of a company’s fiscal year-end (annual data item #199) and net earnings per share (annual data item #57). Stock prices are adjusted for all stock splits and stock dividends occurring during the fiscal year. Earnings per share exclude discontinued operations, extraordinary items, and preferred dividends and are adjusted for stock splits and dividends occurring subsequent to the reporting period. We drop all firm-year observations where the stock price and net earnings per share are (i) smaller than zero, (ii) equal zero, or (iii) have missing values. These requirements lead to a sample size of 195,355 firm-year observations. Further, we only include firms in
our sample that have at the same time at least sixteen consecutive stock price and earnings per share observations available. Since our final regression model contains a one period time lagged stock price as an independent variable, we will lose one of the sixteen observations, and thus, exactly 15 consecutive observations will be left for our final analysis. The chosen length of time series is necessary to obtain reliable regressions results from a dynamic time series model. The structure of our data enables us to estimate time series regressions separately for each firm as well as longitudinal type of panel data regressions. The final sample consists of 59,464 firm-year observations (2,422 individual firms incorporated in the U.S.). Table 1 reports the corresponding descriptive statistics.

3.2 Empirical Models

In order to be able to test our hypotheses empirically, we need to operationalize model (21) in a way that first incorporates condition (20) and second is expressed solely based on observable data. In doing so, we rearrange equation (18) to:

\[ X_{i,t+1}^e = (1 - 2\gamma)X_{i,t}^e + \gamma X_{i,t} \]  

(29)

Further, substitution of (29) into equation (3) yields:

\[ P_{i,t} = a + \gamma b X_{i,t} + b (1 - 2\gamma)X_{i,t-1} \]  

(30)

Now, lagging equation (3) by one period, multiplying it by \((1 - 2\gamma)\), and subtracting this product form equation (30), as well as conduction of some simple algebraic rearrangements leads to:

\[ P_{i,t} = 2\gamma a + \gamma b X_{i,t} + (1 - 2\gamma)P_{i,t-1} \]  

(31)

The model is now expressed entirely in terms of observable variables and therefore empirically testable (Waud, 1968; Lee & Wu, 1988; Maddala & Lahiri, 2009, p. 514). The coefficients of interest can either be identified by a non-linear estimation approach as provided by several software packages or the model can be transformed to:

\[ X_{i,t}^u = \left( X_{i,t} - X_{i,t}^e \right) \]

Further, making use of eq. (20) and substituting \(\phi\) by \(1 - \gamma\) in the former equation as well as rearranging terms, leads to:

\[ X_{i,t+1}^e = (1 - 2\gamma)X_{i,t}^e + \gamma X_{i,t} \]

(31)

See also appendix A1 for a detailed derivation of equation (31).
\[ P_{lt} = \delta_0 + \delta_1 X_{lt} + \delta_2 P_{l,t-1} + u_{lt} \]  

(32)

where \( u_{lt} \) denotes an error term, and be estimated by a linear OLS approach. The coefficients of interests are then given by:

\[ \gamma = (\delta_2 - 1)/-2 \]  

(33)

\[ b = \delta_1/\gamma \]  

(34)

\[ a = \delta_0/2\gamma \]  

(35)

Model (32), however, is restrictive in the sense that it implies only a linear relation between \( \phi \) and \( \gamma \) and not a more general function as denoted in (19). We show solutions for this issue in our robustness tests. In order to strengthen H3 and to visualize the occurring bias of estimated P/E ratios and ERCs from the traditional price model as compared to our extended price model, we additionally estimate the traditional price model where the stock price \( (P_{l,t}) \) is a linear function of current net earnings \( (X_{l,t}) \) per share:

\[ P_{l,t} = a + bX_{l,t} + u_{l,t} \]  

(36)

and where \( u_{l,t} \) denotes an error term, \( a \) an intercept, and \( b \) the slope coefficient.

4. Results

4.1 Primary Regression Results

4.1.1 Results on H1 and H2

In order to provide empirical evidence on H1 and H2, we first run firm-level regressions of model (32) on the full sample of 2,422 individual firms with the aim of determination of the coefficients \( \delta_1 \) and \( \delta_2 \) on the firm-level. We only keep those firms for further analyses for which \( \delta_1 \) and \( \delta_2 \) could be estimated at least at a significance level of 1% (t-value > 2.576). 629 out of 2,422 firms fulfill this condition. Next, according to equation (33) we determine for each of these 629 firms the estimated coefficient \( \gamma \) and partition these firms into broader bins, depending on the magnitude of the estimated \( \gamma \)'s. In particular, we define the following eight bins: \( 0 < \gamma < 0.05; \ 0.05 < \gamma < 0.10; \ 0.10 < \gamma < 0.15; \ 0.15 < \gamma < 0.20; \ 0.20 < \gamma < 0.25; \ 0.25 < \gamma < 0.30; \ 0.30 < \gamma < 0.35; \ 0.35 < \gamma < 0.40 \) based on the empiri-
ical distribution of firm-level estimates of $\gamma$ (see figure 5). In a next step, we run panel regressions of model (32) for each of these eight bins, and determine average estimates for $b$, $\gamma$, $\phi$, $ERC = by$, and P/E ratio $= b\phi$. Regression results are summarized in table 2 and reported in detail in table 3. All slope coefficients from (32) could be estimated significantly at a 1% level.

As predicted, table 2 shows a negative relation between the value-relevant fraction of earnings innovations ($\gamma$) and the persistence of expected earnings ($\phi$). This is consistent with the findings in Freeman & Tse, 1992. However, a comparison of estimated persistence coefficients $\phi$ with $ERC$ magnitudes across the eight bins reveals a negative correlation between the two constructs and at first appearance contradicting findings as compared to prior studies that report a positive association between the two constructs (Kormendi & Lipe, 1987; Collins & Kothari, 1989). Since prior studies (i) do not explicitly distinguish between the marginal P/E ratio, capturing the association between the stock price and expected earnings on the one hand and ERCs, capturing the association between the stock price and unexpected earnings, on the other hand, and (ii) ignore the negative correlation between $\phi$ and $\gamma$ in designing price/return-earnings relations, a positive correlation between earnings persistence and ERC magnitudes has been reported. Our analysis, however, extends prior findings by showing that a positive association between the two constructs only exists within the traditional price model which is based on the restrictive random walk assumption. A simultaneous extension of the traditional price model by (i) the ‘noise in earnings’ assumption and (ii) the ‘negative $\phi$-$\gamma$-relation’ leads to inverted results. These results are consistent with the fact that the direct positive effect of an increase in $\gamma$ on the ERC magnitude outweighs its indirect negative effect via the persistence of earnings ($\phi = f^-(\gamma)$) on the $b$-coefficient ($b = f^+(\phi)$) and in the end on the ERC magnitude: $\gamma \uparrow \Rightarrow \phi \downarrow \Rightarrow b \downarrow \Rightarrow ERC = \gamma(l)b(\uparrow)$. The P/E ratio remains positively associated with the persistence of expected earnings. These results are consistent with H1.
Table 2 also shows a positive correlation between the magnitude of the value-relevant fraction of earnings innovations and ERC magnitudes. Again, the results seem at first glance being contradicting to prior findings reported in Freeman & Tse, 1992. This apparent ‘contradiction’ is likewise attributable to the way how price-earnings relations in prior studies had been designed. Extending the price model by the discussed assumptions reveals that the magnitude of the value-relevant fraction of earnings innovations is positively associated with ERC magnitudes and negatively with P/E ratio magnitudes. These results are consistent with H2.

4.1.2 Results on H3

First, considering results in table 2 reveals that within the earnings persistence interval of $0.63 \leq \phi \leq 0.95$ ($0.65 \leq \phi \leq 0.97$) P/E ratios range between 13.17 and 53.34 (13.46 and 105.06). For the same persistence intervals ERCs range between 7.80 and 2.88 (7.16 and 3.16). These results are consistent with H3.

Second, in order to provide further empirical evidence on H3, results of 485 firm-specific time series regressions of models (32) and (36) are reported in figures 6 and 7. These figures show the distributions of firm-level estimates of the P/E ratios (i) obtained (i) from the traditional price model (eq. (36)) and our extended price model (eq. (32)). Within the traditional price model the P/E ratio equals to the coefficient $b$ (figure 6). Within our extended price model, however, the P/E ratio equals to $b\phi$ (figure 7). We only present results for firms for which these coefficients could be estimated significantly at a level of 1% in equation (36) and for which both slope coefficients of equation (32), $\delta_1$ and $\delta_2$, could be estimated significantly at a 1% level. As a consequence of these restrictions, the distributions shown in figures 6 and 7 are based on the same sample, consisting of 14,623 firm-year observations, including 485 individual firms. For each of these 485 firms the P/E ratios from models (32) and (36) are determined and compared. The comparison shows that the mean P/E ratio estimated from the standard price model (36) (Mean $b = 10.10$) is significantly lower than the P/E ratio (Mean $b\phi = 25.28$) from the extended price model (32). The null hypothesis for a two-sided test of equality of the
two means is strongly rejected ($p$-value = 0.000). These results further strengthen the assertion that the pure random walk model leads to a misspecified price-earnings relation and as a consequence to downward biased P/E ratios. The explicit introduction of (i) the ‘noise in earnings’ hypothesis and the (ii) premise that the magnitude of the value-relevant fraction of earnings innovations is negatively related to earnings persistence into the price model as well as the assumption that (iii) annual earnings are integrated of order one and thus can appropriately be described by an ARIMA$(1,1,0)$ process, leads to price-earnings relations which provide estimates of P/E ratios that equal their theoretically predicted values of 10-60. Considering the distribution reported in figure 7 reveals that most firms have P/E ratios that range between 10 and 60. Figure 8 shows the distribution of the estimated $\gamma$-coeffititents and figure 9 the estimated ERCs for the 485 firms. The ERC magnitudes have a mean of 6.83 (median of 6.10) and range between a minimum of 0.60 and a maximum of 47.24. Most of the firms in our sample exhibit ERCs that range between 3 and 10 (figure 9). This is slightly higher than predicted, but can still be interpreted as consistent with H3.35

4.2 Economic and Practical Implications

The presented results show that our extended price model seems to be a useful approach to describe how investors form expectations about future earnings. It leads to a methodological refinement of the price-earnings relation, as stated in (32), which seems to be ‘better’ in terms of a bias in the estimated P/E ratios/ERCs and the explanatory power compared with traditional price models which are founded on the random walk assumption.

Our findings may have implications for earnings quality research which makes use of the ERC as a proxy. Beginning with Beaver (1968) and Ball Brown (1968) empirical accounting research has established that equity market responses to accounting information. Based on these fundamental results, accounting theory assumes that investors only respond to information that has value implications. If

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35 Non-constant discount rates and/or non-linear negative $\phi$-$\gamma$-relations’ as well as biases caused by the limited number of observations on a firm level can be reasons for the slightly higher firm-level estimates of ERCs.
accounting information would not have value implications, there were no expectation revisions, triggering buy/sell decisions. The ERC is a common measure of investors’ responsiveness to earnings news. Accounting theory further assumes that investors’ responsiveness to earnings is a direct proxy for earnings quality or for earnings informativeness (Dechow et al., 2010, p. 366). If current unexpected earnings are informative to investors, they will cause a forecast revision and will be positively associated with the magnitude of the ERC. The degree of usefulness for investors is measured by the extent of markets reaction, i.e. the higher the price change the higher earnings informativeness and earnings quality, respectively.

However, caution should be exercised when equating informativeness or quality with the extent of security price change. One could also argue that earnings informativeness is high when no expectation revisions occur subsequently to earnings announcements. This view implies that investors’ were well-informed about the value-relevant component of reported earnings prior to the earnings announcement, making expectation revisions in retrospect unnecessarily. In this case the P/E ratio seems to be the adequate coefficient to look at rather than the ERC itself in order to assess earnings quality or earnings informativeness. As can be seen from table 2, if expected earnings have a high persistence, indicating a smooth growth path of its time series, the P/E ratio is high; the ERC, however, is then low. On the other hand, low persistence of expected earnings is associated with high ERCs and low P/E ratios. Our analysis shows that it is important to distinguish between the two concepts of P/E ratios and ERCs in order to fully understand their respective determinants and to receive reliable implications for accounting practice.

5. Additional Analyses

5.1 Non-linear Relation among $\phi$ and $\gamma$

Whether the positive or the negative effect of a change in $\gamma$ on the ERC outweighs, mainly depends on the specific form of the non-linear relationship between $\phi$ and $\gamma$ stated in equation (19), that is on the
magnitude of $\alpha_1$. Figure 1 shows also pathways for $\alpha_1$-values that differ from unity. Figure 10 shows depending on the magnitude of $\alpha_1$ the corresponding pathways for the marginal P/E ratios and the ERCs. In this simulation we exemplarily chose 0.5 for $\alpha_1$’s smaller than unity and 3 for $\alpha_1$’s bigger than unity. Figure 10 reveals that for $\alpha_1 < 1$ (i.e. 0.5) the monotonic negative association among the persistence of expected earnings and ERCs and the monotonic positive association among the magnitude of the value-relevant fraction of earnings innovations remains as in the case of the assumed linear relationship between $\phi$ and $\gamma$. Things can change, however, when $\alpha_1$ is significantly higher than unity, e.g. $\alpha_1 = 3$. In particular, the ERC then is a concave function of earnings persistence and the value-relevant fraction of earnings innovations, respectively. Figure 10 also shows that in the special case of $\alpha_1 = 3$, the ERC increases with increasing persistence of expected earnings within the interval $0 < \phi < 0.65$. Once the threshold of about 0.65 is exceeded the relationship changes and the ERC decreases with further increasing persistence within the interval $0.65 < \phi < 1$.

Since the direction of the association between the ERC and the persistence of expected earnings seems to be a function of $\alpha_1$, it is important to prove whether our assumption made in equation (20) i.e., $\alpha_1 = 1$ holds. Appendix 2 shows that when no assumption is made about the magnitude of $\alpha_1$ the price-earnings relation transform to:

$$P_{it} = \delta_0' + \delta_1'X_{it} + \delta_2'P_{it-1}$$

where $\delta_0' = a[1 - \phi + \gamma]$, $\delta_1' = b\gamma$, and $\delta_2' = (\phi - \gamma)$ and where the individual coefficients are not identifiable anymore. One possible solution to deal with this problem is to estimate equation (37) conditionally on some predefined coefficients. Since it is hard to a priori predefine values for the magnitudes of the value-relevant fraction of an earnings innovation, we rather try in a first step to determine the persistence of expected earnings ($\phi$) out of model (37) and to use those predetermined estimates in a second step for the estimation of (37). What is needed for estimation of (37) but unobservable is the persistence of expected earnings ($\phi$):
\[ X_{t+1} = \beta + \phi X_t \] (38)

Since investors’ expectations are unobservable, equation (38) is not directly testable. However, one way to determine \( \phi \) is to make use of information incorporated in stock prices. Since the valuation model states that the current stock price is a linear function of the unobservable expected earnings (see eq. (3)), the persistence of stock prices must equal the persistence of expected earnings:  

\[ P_t = \lambda + \phi P_{t-1} \] (39)

We thus use equation (39) to determine firm-level estimates of the persistence of expected earnings out of model (37). We subsequently use these predetermined coefficients for empirical estimation of (37). Since Collins & Kothari, 1989 show that both earnings persistence and earnings growth are captured by the slope coefficient of (38), we extend model (39) by controlling for economic growth to:

\[ P_t = \lambda + \phi P_{t-1} + \psi \text{Growth}_{lt} + u_{lt} \] (40)

where \( \text{Growth}_{lt} \) is a time trend variable, taking on the values 1, 2, ..., \( T \), where \( T \) is maximum length of a firm’s time series and where \( u_{lt} \) is an error term. After determination of estimates for \( \phi \) from (40), we conduct two tests to infer conclusions about the magnitude of \( \alpha_1 \) within our sample. First, we run firm-level regressions of (37) and determine the coefficients \( \delta_2' \) which equal \( (\phi - \gamma) \). Making use of equation (A13)\(^{37} \), we substitute \( \gamma \) in \( \delta_2' \) by: \( \gamma = (1 - \phi)^{1/\alpha_1} \). After rearranging terms, we obtain:

\[ \alpha_1 = \frac{\ln(1 - \phi)}{\ln(\phi - \delta_2')} \] (41)

Accordingly to (41), we are now able to determine firm-level estimates of the \( \alpha_1 \)-coefficient and to test whether our initially made assumption of \( \alpha_1 = 1 \) holds or not. Figure 11 shows the empirical distribution of \( \alpha_1 \)-estimates. For the majority of firms in our sample the assumption \( \alpha_1 \leq 1 \) holds. The mean (median) of estimated \( \alpha_1 \)-coefficients amounts to 0.6734 (0.5376) which is far below unity and thus consistent with our assumption made in eq. (20).

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\(^{36}\) The inverse function of (3) equals: \( P_{t-a} = X_{t+1} / \beta \). Thus, the estimation of \( X_{t+1} = \beta + \phi X_t \) and \( P_t = \delta + \phi P_{t-1} \) yields the same slope coefficient, namely \( \phi \) and only differ in estimates of the intercept.

\(^{37}\) (A13) equals to \( \gamma = (1 - \phi)^{1/\alpha_1} \).
Our second test is to run a nonlinear panel regression of equation (A14) under accordingly to (40) pre-determined values of $\phi$:

$$P_{i,t} = a[1 - \phi + (1 - \phi)\alpha_{\alpha_1}] + b(1 - \phi)\alpha_{\alpha_1}X_{i,t} + (\phi - (1 - \phi)\alpha_{\alpha_1})P_{i,t-1} \quad (42)$$

This regression leads to an average $\alpha_1$-estimate of 1.03. Thus, our both tests support the initially made assumption of $\alpha_1 \leq 1$, making it reasonable to assume that ERCs (P/E ratios) are monotonically decreasing (increasing) functions of the persistence of expected earnings, and that ERCs (P/E ratios) are monotonically increasing (decreasing) functions of the value-relevant portion of earnings innovations within price models.

5.2 Alternative Price-Earnings Specifications

In order to strengthen our results, we also run two alternative regressions of the price-earnings specification. As a first additional test, we run our analysis based on the deflated price model:

$$\frac{P_{i,t}}{P_{i,t-1}} = \delta_0 + \delta_1 \frac{X_{i,t}}{P_{i,t-1}} + \delta_2 + u_{i,t} \quad (43)$$

Our second test is based on a logarithmic price model:

$$\ln(P_{i,t}) = \delta_0 + \delta_1 \ln(X_{i,t}) + \delta_2 \ln(P_{i,t-1}) + u_{i,t} \quad (44)$$

A summary of conducted regression results is presented in table 4. The results of specifications (43) and (44) are still consistent with H1 to H3. In particular, both models reveal the inverted results between P/E ratios/ERCs and $\phi$ and $\gamma$, respectively, being consistent with H1 and H2. However, the deviations between predicted and estimated P/E ratio and ERC magnitudes rise as compared to the original ‘level-form’ of the price model presented in equation (32), challenging the consistency with H3. This is mainly due to the fact that the time series properties of scaled/deflated or logarithmic annual earnings differ from their original time series properties. In particular, transformed time series of earnings often do not follow an ARIMA(1,1,0) model anymore, changing the expectations of P/E ratio and ERC magnitudes in theory. The same applies to other transformations as return models and differenced price models. Since, mathematical transformations of the original price model change the empirical time series properties of annual earnings and stock prices, expectations regarding the magnitudes
of P/E ratios and ERCs within those transformed models must be adjusted in order to obtain consistency between theoretically predicted and empirically estimated P/E ratios and ERCs. In summary, mathematical/empirical transformations of the traditional ‘level-form’ price model changes the time series properties of stock prices and accounting earnings, making revisions of the theoretical P/E ratio and ERC magnitudes necessary.

5.3 Further Robustness Tests

In order to validate our results, we perform some further robustness tests. First, we run model (32) depending on different bin partitions then the reported in tables 2 and 3 (e.g. based on 0.1-γ-steps). The results remain similar. Second, we also perform our analysis based on shorter time series requirements. Instead of 15 consecutive observations, we also use samples with 10 consecutive observations. The results remain unchanged. We further conduct our analysis based on different time periods. Instead of using data for the whole available period of 1962-2013, we test our model also based on the following time periods: (i) 1962-1987, (ii) 1987-2013 (sample division in half), and (iii) 2003-2013 (last 10 years). The results remain unchanged. Since it has been shown that losses have different ‘information contents’ to investors (Hayn, 1995), we also run our analysis on samples including loss observations. Including losses into the analysis and controlling for its effect by introduction of loss-dummy-variables slightly blurs the results. However, the results remain still consistent with H1 to H3.

6. Conclusion

We investigate the associations between earnings innovations, persistence of expected earnings, and the magnitudes of price-to-earnings ratios (P/E ratios) and earnings response coefficients (ERCs), respectively. We extend the traditional price model by a simultaneous incorporation of three assumptions: (i) earnings are only partially value-relevant and investors are sophisticated enough to extract this portion and to price it adequately; (ii) the persistence of expected earnings is negatively correlated with the value-relevant magnitude of earnings innovations; (iii) an ARIMA(1,1,0) process is appropri-
ate in describing the time series properties of longitudinal annual earnings data. We show that these extensions have significant implications for both the above mentioned associations and the magnitudes of P/E ratios and ERCs, respectively.

In particular, the introduction of the ‘noise in earnings’ hypothesis into the price model, reveals that the ERC is the product of the value-relevant magnitude of earnings innovations and the P/E ratio. Since on the one hand the persistence of expected earnings is negatively correlated with the value-relevant magnitude of earnings innovations (‘negative $\phi$-$\gamma$-relation’) and on the other hand the magnitude of the P/E ratio is positively correlated with the persistence of expected earnings, two opposite effects exist determining the magnitude of the ERC. As a consequence, it is an empirical question to ascertain which effect dominates. Our empirical findings show that the positive effect of an increase in the value-relevant magnitude of earnings innovations outweighs the negative effect of its increase on the persistence of expected earnings and thus on the magnitude of the P/E ratio and in the end on the ERC magnitude. Because of this relation, ERC magnitudes are negatively associated with the persistence of expected earnings and positively with the magnitude of the value-relevant fraction of earnings innovations, extending prior findings derived from traditional price/return models (Kormendi & Lipe, 1987; Collins & Kothari, 1989; Collins & Salatka, 1993; Freeman & Tse, 1992). P/E ratios, however, remain to be positively associated with the persistence of expected earnings and negatively with magnitudes of the value-relevant fraction of earnings innovations.

Further, since an ARIMA(1,1,0) process seems to be more appropriate in describing the properties of longitudinal time series of accounting earnings as compared to a random walk, expectations regarding the magnitudes of P/E ratios and ERCs change and has to be revised. In particular, assuming a discount rate of 15%, theoretical P/E ratios range from 10 to 60 and ERCs from 0 to 6 within the earnings persistence interval of 0.6 to 1. Our empirical findings are consistent with these predictions, closing the gap between theoretically expected and empirically estimated P/E ratio and ERC magnitudes, respectively. Our results have also important implications for earnings quality/informativeness research.
We provide preliminary evidence that earnings informativeness is high when P/E ratios (ERCs) are high (low). Finally, our extension of the price model leads to a methodological refinement of the price-earnings relation. As a consequence, (i) estimated P/E ratios and ERC are unbiased and amount to their predicted magnitudes, and (ii) the explanatory power of the price-model significantly increases.
References


Figure 1: Possible negative Relationships among $\phi$ and $\gamma$

Different Pathways among $\phi$ and $\gamma$, depending on the magnitude of $\alpha_1$ derived from equation (19): $\phi = \alpha_0 - \gamma^{\alpha_1}$, where $\alpha_0 = 1$.

Figure 2: The Relations between P/E Ratio and ERC Magnitudes and the Persistence of Expected Earnings

Hypothetical values for P/E ratios and ERCs obtained from equations (25) and (26), assuming a discount rate of 15%.
Figure 3: The Relations between P/E Ratio and ERC Magnitudes and the Value-Relevant Magnitude of Earnings Innovations

Hypothetical values for P/E ratios and ERCs obtained from equation (28), assuming a discount rate of 15%.

Figure 4: Freeman & Tse Relation between ERC Magnitudes and the Value-Relevant Magnitude of Earnings Innovations

Hypothetical values derived from the model: $UR = a_0 + a_1 \arctan(a_2 UE_i) + e_i$ where $UR$ denotes unexpected returns and $UE_i$ unexpected earnings and where $a_0 = 0.008$, $a_1 = 0.04$, and $a_2 = 310$ (see Fremman & Tse, 1992, p. 190 and p. 195 table 1).
Figure 5: Empirical Distribution of estimated $\gamma$ and Bin Partition

Empirical distribution of significant firm-level estimates of $\gamma$-coefficients based on model (32) (N = 629).

Table 1: Descriptive Statistics

<table>
<thead>
<tr>
<th>Variable</th>
<th>Mean</th>
<th>Std. Dev.</th>
<th>Min.</th>
<th>0.25</th>
<th>Median</th>
<th>0.75</th>
<th>Max</th>
<th>N</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P$</td>
<td>29.82</td>
<td>20.78</td>
<td>2.57</td>
<td>15.75</td>
<td>25.13</td>
<td>38</td>
<td>120.88</td>
<td>59,464</td>
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<tr>
<td>$X$</td>
<td>2.26</td>
<td>1.66</td>
<td>0.10</td>
<td>1.13</td>
<td>1.88</td>
<td>2.90</td>
<td>9.35</td>
<td>59,464</td>
</tr>
<tr>
<td>Time Series Length</td>
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<td>7.42</td>
<td>16.00</td>
<td>18.00</td>
<td>21.00</td>
<td>28.00</td>
<td>51.00</td>
<td>21,949</td>
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</table>


Table 2: Summary of Panel Regression Results from Model (32)

<table>
<thead>
<tr>
<th>Bin</th>
<th>$\gamma$-Range</th>
<th>Firm FE</th>
<th>Firm FE &amp; Year FE</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.00-0.05</td>
<td>$\gamma$</td>
<td>$P/E$</td>
</tr>
<tr>
<td>3</td>
<td>0.10-0.15</td>
<td>0.03</td>
<td>108.22</td>
</tr>
<tr>
<td>4</td>
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<td>0.13</td>
<td>30.37</td>
</tr>
<tr>
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<td>0.25-0.30</td>
<td>0.20</td>
<td>27.62</td>
</tr>
<tr>
<td>7</td>
<td>0.30-0.35</td>
<td>0.24</td>
<td>21.51</td>
</tr>
<tr>
<td>8</td>
<td>0.35-0.40</td>
<td>0.29</td>
<td>20.63</td>
</tr>
</tbody>
</table>

It applies that: $\gamma = (\delta_2 - 1)/2$, $b = \delta_1/\gamma$, and $\phi = 1 - \gamma$. $P/E = b\phi$ and $ERC = b\gamma$. FE = fixed effects.
Table 3: Panel Regression Results from Model (32)

<table>
<thead>
<tr>
<th>Bin</th>
<th>𝑝-Range</th>
<th>( P_{i,t} = \delta_0 + \delta_1 X_{i,t} + \delta_2 P_{i,t-1} + u_{i,t} )</th>
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</thead>
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<td></td>
<td></td>
<td>Firm Fixed Effects</td>
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<td></td>
<td>Coeff</td>
</tr>
<tr>
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<td>0.00-0.05</td>
<td>( \delta_0 )</td>
</tr>
<tr>
<td></td>
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<tr>
<td></td>
<td></td>
<td>( \delta_2 )</td>
</tr>
<tr>
<td>2</td>
<td>0.05-0.10</td>
<td>( \delta_0 )</td>
</tr>
<tr>
<td></td>
<td></td>
<td>( \delta_1 )</td>
</tr>
<tr>
<td></td>
<td></td>
<td>( \delta_2 )</td>
</tr>
<tr>
<td>3</td>
<td>0.10-0.15</td>
<td>( \delta_0 )</td>
</tr>
<tr>
<td></td>
<td></td>
<td>( \delta_1 )</td>
</tr>
<tr>
<td></td>
<td></td>
<td>( \delta_2 )</td>
</tr>
<tr>
<td>4</td>
<td>0.15-0.20</td>
<td>( \delta_0 )</td>
</tr>
<tr>
<td></td>
<td></td>
<td>( \delta_1 )</td>
</tr>
<tr>
<td></td>
<td></td>
<td>( \delta_2 )</td>
</tr>
<tr>
<td>5</td>
<td>0.20-0.25</td>
<td>( \delta_0 )</td>
</tr>
<tr>
<td></td>
<td></td>
<td>( \delta_1 )</td>
</tr>
<tr>
<td></td>
<td></td>
<td>( \delta_2 )</td>
</tr>
<tr>
<td>6</td>
<td>0.25-0.30</td>
<td>( \delta_0 )</td>
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<td>( \delta_1 )</td>
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<tr>
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<td>( \delta_1 )</td>
</tr>
<tr>
<td></td>
<td></td>
<td>( \delta_2 )</td>
</tr>
<tr>
<td>8</td>
<td>0.35-0.40</td>
<td>( \delta_0 )</td>
</tr>
<tr>
<td></td>
<td></td>
<td>( \delta_1 )</td>
</tr>
<tr>
<td></td>
<td></td>
<td>( \delta_2 )</td>
</tr>
</tbody>
</table>

* *, **, *** indicate statistical significance at 10%, 5%, and 1% level, respectively. The t-Statistic is calculated using a two-tailed test and based on robust standard errors clustered by firm. FE = fixed effects.
Figure 6: Distribution of P/E Ratios ($b$) estimated from the Price Model (36)

![Histogram showing distribution of P/E Ratios ($b$). Mean 10.10; Median 9.22; Min. 0.62; Max 65.34; Std. Dev. 5.57; N 14,623 (485 firms).]

Figure 7: Distribution of P/E Ratios ($b\phi$) estimated from the Price Model (32)

![Histogram showing distribution of P/E Ratios ($b\phi$). Mean 25.28; Median 20.93; Min. 4.09; Max 654.93; Std. Dev. 26.49; N 14,623 (485 firms).]

38 Two P/E ratio observations are larger than 120 and not documented graphically in figure 7.
Figure 8: Distribution of $\gamma$-coefficients estimated from the Price Model (32)

Mean 0.230; Median 0.234; Min. 0.003; Max 0.396; Std. Dev. 0.064; N 14,623 (485 firms)

Figure 9: Distribution of ERCs ($b\gamma$) estimated from the Price Model (32)

Mean 6.83; Median 6.10; Min. 0.60; Max 47.24; Std. Dev. 3.92; N 14,623 (485 firms)
Figure 10: Possible Pathways of ERCs and P/E Ratios for different $\alpha_1$-Magnitudes

Figure 11: Empirical Distribution of estimated $\alpha_1$-Magnitudes from (41)

Mean: 0.6734 Min: 0.0823 Median: 0.5376 Max: 8.2204 Std. Dev.: 0.600 N = 411
Table 4: Summary of Panel Regression Results from Models (43) and (44)

\[
\frac{P_{it}}{P_{i,t-1}} = \delta_0 + \delta_1 \frac{X_{it}}{P_{i,t-1}} + \delta_2 + u_{it}
\]

<table>
<thead>
<tr>
<th>Bin</th>
<th>Range</th>
<th>( \gamma )</th>
<th>( \phi )</th>
<th>( b )</th>
<th>( P/E )</th>
<th>ERC</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
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<td>0.95</td>
<td>46.39</td>
<td>44.02</td>
<td>2.37</td>
</tr>
<tr>
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<td>0.14</td>
<td>0.86</td>
<td>23.39</td>
<td>20.21</td>
<td>3.17</td>
</tr>
<tr>
<td>3</td>
<td>0.10-0.15</td>
<td>0.16</td>
<td>0.84</td>
<td>25.33</td>
<td>21.33</td>
<td>3.99</td>
</tr>
<tr>
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<td>25.29</td>
<td>21.39</td>
<td>3.90</td>
</tr>
<tr>
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<td>0.20-0.25</td>
<td>0.20</td>
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<td>24.15</td>
<td>19.34</td>
<td>4.80</td>
</tr>
<tr>
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<td>0.75</td>
<td>23.93</td>
<td>17.99</td>
<td>5.94</td>
</tr>
<tr>
<td>7</td>
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<td>0.28</td>
<td>0.72</td>
<td>25.51</td>
<td>18.29</td>
<td>7.22</td>
</tr>
</tbody>
</table>

\[
\ln(P_{it}) = \delta_0 + \delta_1 \ln(X_{it}) + \delta_2 \ln(P_{i,t-1}) + u_{it}
\]

<table>
<thead>
<tr>
<th>Bin</th>
<th>Range</th>
<th>( \gamma )</th>
<th>( \phi )</th>
<th>( b )</th>
<th>( P/E )</th>
<th>ERC</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
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<td>0.15</td>
<td>0.85</td>
<td>28.88</td>
<td>24.58</td>
<td>4.30</td>
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<td>2</td>
<td>0.05-0.10</td>
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<td>0.43</td>
<td>0.57</td>
<td>31.00</td>
<td>17.71</td>
<td>13.29</td>
</tr>
</tbody>
</table>

It applies that: \( \gamma = (\delta_2 - 1)/\delta_1 \), \( b = \delta_1/\gamma \), and \( \phi = 1 - \gamma \). \( P/E = b \phi \) and \( ERC = b \gamma \). Panel regressions have been run including year and firm fixed effects.
Appendix A1: Operationalization of Model (21) for Empirical Estimation under the Assumptions of $\alpha_0 = 1$ and $\alpha_1 = 1$.

Consider the following valuation model where the stock price is a linear function of expected one-period ahead future net earnings:

$$ P_{t,t} = a + bX_{i,t+1} $$ (A1)

Assume that investors’ expectations formation is given by:

$$ X_{i,t+1}^e = \phi X_{i,t}^e + \gamma X_{i,t}^u $$ (A2)

and that a negative relation exist among $\phi$ and $\gamma$, given by:

$$ \phi = 1 - \gamma $$ (A3)

Substitution of $\phi$ in (A2) by (A3) and $X_{i,t}^u$ in (A2) by $X_{i,t} - X_{i,t}^e$, leads to:

$$ X_{i,t+1}^e = (1 - \gamma)X_{i,t}^e + \gamma(X_{i,t} - X_{i,t}^e) $$ (A4)

Rearranging terms yields:

$$ X_{i,t+1}^e = (1 - 2\gamma)X_{i,t}^e + \gamma X_{i,t} $$ (A5)

Substitution of $X_{i,t+1}^e$ in (A1) by (A5), leads to:

$$ P_{i,t} = a + b(1 - 2\gamma)X_{i,t}^e + b\gamma X_{i,t} $$ (A6)

Now, lagging (A1) by one period and multiplying it by $(1 - 2\gamma)$, provides:

$$ (1 - 2\gamma)P_{i,t-1} = (1 - 2\gamma)a + (1 - 2\gamma)bX_{i,t}^e $$ (A7)

Subtracting (A7) from (A6), leads to a cancellation of $b(1 - 2\gamma)X_{i,t}^e$ and after rearrangement of terms to the expression:

$$ P_{i,t} = 2\gamma a + b\gamma X_{i,t} + (1 - 2\gamma)P_{i,t-1} $$ (A8)

Appendix A2: Operationalization of Model (21) for Empirical Estimation under the Assumption of $\alpha_0 = 1$.

Again we start with the valuation model (A1) and assume that investors’ expectations formation is given by (A2). However, we now do not make any assumption about the magnitudes of $\alpha_1$.

$$ \phi = 1 - \gamma^{\alpha_1} $$ (A9)
In this more general case the transformation into the empirical model either leads to nonlinearities in
the coefficients or not identifiable coefficients. Substitution of (A2) in (A1) and rearranging terms,
leads to:

\[ P_{i,t} = a + b(\phi - \gamma)X_{t,t}^e + byX_{i,t} \]  \hfill (A10)

Now, lagging (A1) by one period and multiplying it by \((\phi - \gamma)\), provides:

\[ (\phi - \gamma)P_{i,t-1} = (\phi - \gamma)a + b(\phi - \gamma)bX_{i,t}^e \]  \hfill (A11)

Subtracting (A11) from (A10), leads to a cancellation of \(b(\phi - \gamma)bX_{i,t}^e\) and after rearrangement of
terms to the expression:

\[ P_{i,t} = a[1 - \phi + \gamma] + byX_{i,t} + (\phi - \gamma)P_{i,t-1} \]  \hfill (A12)

or rearranging (A3) to:

\[ \gamma = (1 - \phi)^{1/\alpha_1} \]  \hfill (A13)

and substitution of (A13) in (A12) leads to:

\[ P_{i,t} = a\left[1 - \phi + (1 - \phi)^{1/\alpha_1}\right] + b(1 - \phi)^{1/\alpha_1}X_{i,t} + (\phi - (1 - \phi)^{1/\alpha_1})P_{i,t-1} \]  \hfill (A14)

Empirical estimation of either (A12) or (A13) faces the problem that the individual coefficients are not
identifiable.